

Degrees of categoricity and spectral dimension

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Abstract

© 2018 The Association for Symbolic Logic. A Turing degree d is the degree of categoricity of a computable structure S if d is the least degree capable of computing isomorphisms among arbitrary computable copies of S . A degree d is the strong degree of categoricity of S if d is the degree of categoricity of S , and there are computable copies A and B of S such that every isomorphism from A onto B computes d . In this paper, we build a c.e. degree d and a computable rigid structure M such that d is the degree of categoricity of M , but d is not the strong degree of categoricity of M . This solves the open problem of Fokina, Kalimullin, and Miller [13]. For a computable structure S , we introduce the notion of the spectral dimension of S , which gives a quantitative characteristic of the degree of categoricity of S . We prove that for a nonzero natural number N , there is a computable rigid structure M such that 0 is the degree of categoricity of M , and the spectral dimension of M is equal to N .

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Keywords

categoricity spectrum, computable categoricity, degree of categoricity, rigid structure

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